OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2603(A)

Pure Mathematics 3

Section A

Tuesday **10 JUNE 2003** Afternoon 1 hour 20 minutes

Additional materials:
- Answer booklet
- Graph paper
- MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

NOTE

- This paper will be followed by **Section B: Comprehension.**

This question paper consists of 4 printed pages.

HN/4

[Turn over

1 (a) Find the first four terms of the binomial expansion of $(1-3x)^{-2}$. [4]

(b) Express $\dfrac{-3-x}{(x^2+1)(x-1)}$ in the form $\dfrac{Ax+B}{x^2+1}+\dfrac{C}{x-1}$, giving the values of A, B and C. [4]

(c) Find $\int x\cos 2x \, dx$. [4]

(d) Given that $x^2+y^2=y$, show that $\dfrac{dy}{dx}=\dfrac{2x}{1-2y}$. [4]

[Total 16]

2 Water drains from a container so that the height h metres of the water above the base after t seconds satisfies the differential equation

$$\frac{dh}{dt}=-ah,$$

where a is a positive constant. Initially, the height of the water is 2 metres.

(i) Verify that $h=2e^{-at}$ satisfies both the differential equation and the initial condition. [3]

(ii) It takes 10 seconds for the height to drop to 1 metre. Find a. [3]

In another container, the height H metres of the water varies according to the differential equation

$$\frac{dH}{dt}=-b\sqrt{H},$$

where b is a positive constant. The initial height of the water is 1 metre.

(iii) Using integration, show that $\sqrt{H}=1-\frac{1}{2}bt$. [4]

(iv) Given that it takes 10 seconds for the height to drop to a half of its initial value, find b.

Hence find how long it takes for the height to drop from 1 metre to zero. [5]

[Total 15]

3 **(a)** A curve has parametric equations

$$x = \cos\theta,\ \ y = \cos 2\theta, \qquad 0 \leqslant \theta \leqslant \pi.$$

(i) Show that $\dfrac{dy}{dx} = 4\cos\theta$. [4]

(ii) Find the cartesian equation of the curve. Sketch the curve. [3]

(b) Fig. 3 shows the curve with parametric equations

$$x = \cos\theta,\ \ y = \cos 3\theta, \qquad 0 \leqslant \theta \leqslant \pi.$$

The curve cuts the x-axis at the points A, O and B.

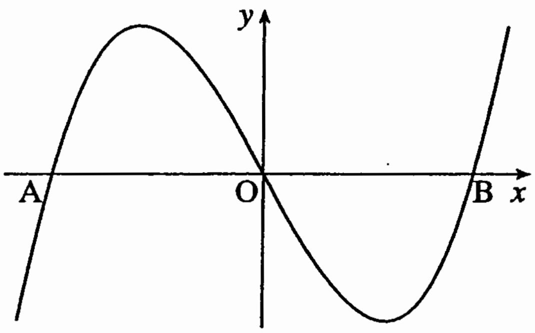

Fig. 3

(i) Find the coordinates of the points A and B. [4]

(ii) By first expanding $\cos(2\theta + \theta)$, show that

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta.$$

Hence write down the cartesian equation of the curve. [4]

[Total 15]

4 As part of a sculpture, an artist erects a flat triangular sheet ABC in his garden. The vertices are attached to vertical poles DA, EB and FC. The coordinate axes Ox and Oy are horizontal, and Oz is vertical. The coordinates of the triangle are A(2, 0, 2), B(−2, 0, 1) and C(0, 4, 3), with units in metres (see Fig. 4).

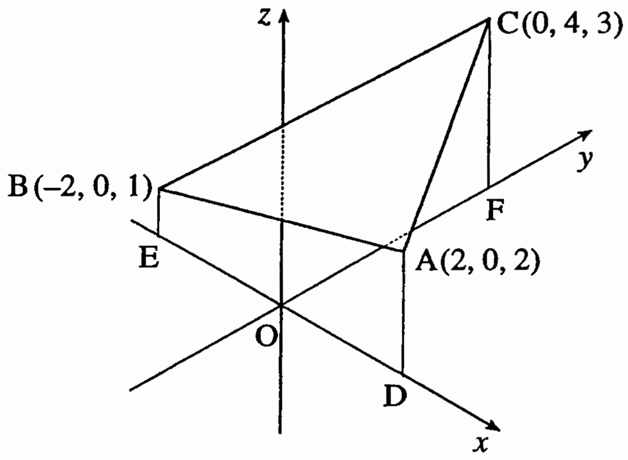

Fig. 4

(i) Find the length of the side AC. [2]

(ii) Find the scalar product $\overrightarrow{AB}.\overrightarrow{AC}$, and the angle BAC. [4]

(iii) Show that $\begin{pmatrix} 2 \\ 3 \\ -8 \end{pmatrix}$ is perpendicular to the lines AB and AC.

Hence find the cartesian equation of the plane ABC. [5]

(iv) The artist decides to erect another vertical pole GH based at the point G(1, 1, 0). Calculate the height of the pole if H is to lie in the plane ABC. [3]

[Total 14]

Electing a representative parliament

Background

The results of the 2001 UK General Election for the Westminster parliament are summarised in the first three columns of Table 1. The fourth column gives the number of members of parliament (MPs) that the parties would have had if they had been allocated MPs in proportion to the total votes cast.

Party	% of national vote	MPs elected	MPs in proportion
Conservative	31.7	166	209
Labour	40.7	412	268
Liberal Democrat	18.3	52	121
Others	9.3	29	61
Total	100.0	659	659

Table 1

There are considerable differences between the numbers in the last two columns. Every recent election has produced a similar discrepancy. This has led to calls for a change in the voting system to ensure that the make-up of parliament is more representative of the votes cast, and so of the will of the electorate.

Voting systems

There are many possible voting systems. Information on those not discussed in this article is available from The Electoral Reform Society (www.electoral-reform.org.uk).

First Past The Post

In the present system in the UK, you have just one vote for a local MP. The candidate with the largest number of votes is the winner. However you voted, your MP is your personal representative in parliament at Westminster.

This system is given the misleading description of *First Past The Post*. (There is no "post" that the winner has to pass; all that is necessary is to receive more votes than each of the other candidates.) It is also used, among other places, in Canada, the USA and India.

Party List

The *Party List* system is widely used in the rest of Europe. Electors vote for a party rather than an individual candidate. Each party is then allocated a number of MPs in proportion to the votes it receives. There are, however, two fundamental problems with such a system.

The first is that the people in any constituency are no longer voting for their MP who will represent them. Instead they are voting for a distant party who will then impose a member on them.

The other problem is that such a system gives party leaders the ability to appoint their friends to parliament. A change from First Past The Post to this system would take power out of the hands of the people and give it to the party leaders.

Allowing two votes

While the Party List system does ensure strict proportionality, many people feel that this does not justify the loss of accountability that goes with it.

This article looks at the consequences of modifying the First Past The Post system by allowing voters two votes, first choice and second choice. Two possible schemes are considered.

In nearly all constituencies the vast majority of the votes are shared between three candidates. The rest of this article is based on the simplifying assumption that there are in fact exactly three candidates, A, B and C. Where percentages are given, they are percentages of votes cast; those choosing not to vote are ignored. It is also assumed that all those voting use both their votes.

With three candidates it is possible, indeed common, for a candidate to be elected on less than 50% of the votes cast. A typical result might be:

A 34%, B 36%, C 30%.

Candidate B is elected even though 64% of the electors voted for someone else.

It is clearly desirable for the person elected to have some support from at least 50% of those voting. Since this cannot always be achieved with only a single vote, it is appropriate to look at the consequences of people having two votes.

The rest of this article considers the large number of cases where no candidate achieves 50% of the first choice votes. It is assumed that any candidate who does in fact receive 50% of the first choice votes is elected.

Scheme 1: Two votes, equally weighted

What would happen if electors were required to give a second choice as well? Three different scenarios are now considered. Two of these represent extreme situations (the bounds of the problem), the third a typical middle-of-the-road situation.

Scenario 1

In this scenario, there is bitter rivalry between candidates A and B. All those who give A as their first vote give C as their second; similarly all those who give their first vote to B prefer C to A for their second. First choice voters for C divide equally between A and B for their second choices.

This gives rise to the percentages of first and second choice votes, denoted by f and s, in Table 2. The final column gives a total for each candidate, with no distinction between first and second choice votes.

Scenario 1	1st Choice, f	2nd Choice, s	$f + s$
A	34	15	49
B	36	15	51
C	30	70	100

Table 2

In this extreme situation C emerges as the clear winner. This outcome is confirmed in Table 3 where the particular numbers used in the example are replaced by letters, a, b and c representing the percentages of first choice votes for A, B and C respectively. ($a, b, c > 0$.)

Scenario 1	1st Choice, f	2nd Choice, s	$f+s$
A	a	$50 - \frac{1}{2}(a+b)$	$50 + \frac{1}{2}a - \frac{1}{2}b$
B	b	$50 - \frac{1}{2}(a+b)$	$50 - \frac{1}{2}a + \frac{1}{2}b$
C	$c = 100 - (a+b)$	$a+b$	100

Table 3

This gives the somewhat surprising result that, since

$$\text{both} \quad 50 + \tfrac{1}{2}a - \tfrac{1}{2}b < 100 \quad \text{and} \quad 50 - \tfrac{1}{2}a + \tfrac{1}{2}b < 100,$$

C must inevitably be the winner in this situation. Everyone has voted for C, either as first or second choice, resulting in an unbeatable total of 100.

This is obviously an extreme case. Perhaps both A and B have spent so long being rude to one another that their supporters have seen C as the only possible alternative.

Scenario 2

In the second scenario, the second choice votes are always spread equally between the remaining two candidates. Thus out of the 34% whose first choice is A, 17% give B as their second choice and 17% give C. The outcomes are given in Table 4.

Scenario 2	1st Choice, f	2nd Choice, s	$f+s$
A	34	33	67
B	36	32	68
C	30	35	65

Table 4

In this case the three candidates are ranked in the same order as they were on first choice votes: B the winner, then A and then C. However their totals are much less spread out.

This can be explained by looking at the general case, shown in Tables 5 and 6.

Scenario 2	1st Choice, f	2nd Choice, s	$f+s$
A	a	$\frac{1}{2}(b+c)$	$a + \frac{1}{2}b + \frac{1}{2}c$
B	b	$\frac{1}{2}(a+c)$	$\frac{1}{2}a + b + \frac{1}{2}c$
C	$c = 100 - (a+b)$	$\frac{1}{2}(a+b)$	$\frac{1}{2}a + \frac{1}{2}b + c$

Table 5

The final column in Table 5 can alternatively be written as shown in Table 6.

$f+s$
$50 + \frac{1}{2}a$
$50 + \frac{1}{2}b$
$50 + \frac{1}{2}c$

Table 6

It follows that, in this situation, the ranking of the candidates must be the same as before but the spread of their totals must be half of that obtained by counting only first choice votes. The result is the same but it looks closer.

Scenario 3

Neither of the outcomes in Scenarios 1 and 2 seems particularly satisfactory. In the first, C seems to be unduly favoured, and in the second, close results would seem inevitable. However they are both extreme cases. Scenario 3 represents a more realistic situation, somewhere between the two. There is still some reluctance among the supporters of A and B to vote for the other but it is not as extreme as in Scenario 1.

In a typical example of this scenario, voters make the following first and second choices.

	Votes (%)
A first, B second	14
A first, C second	20
B first, A second	14
B first, C second	22
C first, A second	15
C first, B second	15

Table 7

The outcome is shown in Table 8. In this case, candidate C wins.

Scenario 3	1st Choice, f	2nd Choice, s	$f+s$
A	34	29	63
B	36	29	65
C	30	42	72

Table 8

Scheme 2: Two votes, unequally weighted

It can be argued that allowing equal weight to first and second choice votes is intrinsically unfair, and that Scheme 1 treats the third candidate, C, too favourably.

An alternative system is to weight the two votes, counting 2 for a first choice and 1 for a second choice. While this gives the same overall outcomes in the three scenarios, the undesirable features are much less evident, as can be seen from Tables 9, 10 and 11.

Scenario 1	1st Choice, f	2nd Choice, s	$2f+s$
A	34	15	83
B	36	15	87
C	30	70	130

Table 9

Scenario 2	1st Choice, f	2nd Choice, s	$2f+s$
A	34	33	101
B	36	32	104
C	30	35	95

Table 10

Scenario 3	1st Choice, f	2nd Choice, s	$2f+s$
A	34	29	97
B	36	29	101
C	30	42	102

Table 11

Scheme 2 is exactly equivalent to giving voters one positive vote for their first choice and one negative vote for their third choice, i.e. the candidate they don't want. This is shown for scenario 3 in Table 12.

Scenario 3	1st Choice, f Positive vote	2nd Choice, s No vote	3rd Choice, t Negative vote	$f-t$
A	34	29	37	–3
B	36	29	35	+1
C	30	42	28	+2

Table 12

The figures in the last column in Table 12 are exactly 100 less than those in the last column of Table 11. Note that

$$f - t = 2f + s - 100.$$

Since the mean value of $(f - t)$ is zero, the winner must have a positive score. Thus you cannot win under this system unless more than 50% of electors vote for you, either as first or second choice.

Conclusion

This article has looked at two schemes in which people have two votes. For a number of reasons Scheme 1 was shown to be unsatisfactory. By contrast Scheme 2 seems to accord with natural justice.

Scheme 2 clearly makes it easier for a third party candidate to get elected than in First Past The Post, but would it result in the number of MPs per party being more nearly proportional to the votes cast?

The answer depends on how voters use their two votes. A commonly heard complaint under the present scheme is "*I would like to vote for the .... party candidate but it would just be a wasted vote*". It could well be that many people would give one of their two votes to one of the smaller parties. In that case the smaller parties could expect to get more MPs elected.

For Examiner's Use

1 The table below has the same format as Table 1 but the figures in it refer to the 1945 General Election. Complete the missing entries. [2]

Party	% of national vote	MPs elected	MPs in proportion
Conservative	39.7	210	
Labour	47.7		
Liberal	9.0	12	58
Others	3.6	25	
Total	100.0	640	640

2 The table below shows the first choice votes in an election in which Scenario 1 applies. Thus all those who give A their first choice vote give C their second choice vote. Similarly all those who give B their first choice vote give C their second choice vote. Half of those who give C their first choice vote give their second choice vote to A and the other half give their second choice vote to B. The votes are then added according to Scheme 1.

Complete the table. [2]

Scenario 1	1st Choice, f	2nd Choice, s	$f + s$
A	46		
B	48		
C	6		

For Examiner's Use

3 Starting with Table 5, justify the entries in Table 6. [3]

Scenario 2	1st Choice, f	2nd Choice, s	$f+s$
A	a	$\frac{1}{2}(b+c)$	$a+\frac{1}{2}b+\frac{1}{2}c$
B	b	$\frac{1}{2}(a+c)$	$\frac{1}{2}a+b+\frac{1}{2}c$
C	$c=100-(a+b)$	$\frac{1}{2}(a+b)$	$\frac{1}{2}a+\frac{1}{2}b+c$

Table 5

$f+s$
$50+\frac{1}{2}a$
$50+\frac{1}{2}b$
$50+\frac{1}{2}c$

Table 6

....

....

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4 Justify the statement

$$f-t=2f+s-100$$

in line 167. [3]

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For Examiner's Use

5 In the text, Table 10 illustrates a particular case of Scenario 2 under Scheme 2. The situation is generalized in the table below.

Scenario 2	1st Choice, f	2nd Choice, s	$2f + s$
A	a		
B	b		
C	c		

(i) Complete the table. [2]

(ii) Compare the outcome with the First Past The Post system, considering both the order and the spread of the results. [3]

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Mark Scheme

SECTION A

1(a) $(1-3x)^{-2} =$ $1+(-2)(-3x)+\frac{(-2)(-3)}{1\times 2}(-3x)^2+\frac{(-2)(-3)(-4)}{1\times 2\times 3}(-3x)^3+...$ $= 1 + 6x + 27x^2 + 108x^3 + \ldots$	M1 B1 B1 B1 [4]	Four correct binomial coefficients as shown (allow one slip) s.o.i. $+ 6x$ $+ 27x^2$(Allow this B1 only, from the use of $(3x)^2$ instead of $(-3x)^2$) $+ 108x^3$
(b) $\frac{-3-x}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1}+\frac{C}{x-1}.$ $\Rightarrow -3 - x = (Ax + B)(x - 1) + C(x^2 + 1)$ $x = 1 \Rightarrow -4 = 2C \Rightarrow C = -2$ coeff^t of x^2: $0 = A + C \Rightarrow A = 2$ constants: $-3 = -B + C \Rightarrow B = 1$	M1 A1 A2,1 or 0 [4]	Equating the numerators s.o.i. If the brackets round Ax+B are omitted allow M1 only if the equations involving A and B are correct. For any one correct equation s.o.i. Deduct 1 for each incorrect value of A,B or C
(c) $\int x\cos 2x dx = \int x\frac{d}{dx}(\frac{1}{2}\sin 2x)dx$ $= \frac{1}{2}x\sin 2x - \int\frac{1}{2}\sin 2x dx$ $= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$ ~~Other correct forms are possible e.g. ¼ x sin 2x + ¼ cos^{2}2x + c etc.~~	M1 A1 A1f.t. A1www [4]	Using the method of integration by parts with $u=x$ and $dv/dx=\cos 2x$ leading to 2 terms. $\frac{1}{2}x\sin 2x$ $-\int\frac{1}{2}\sin 2x dx$ f.t. their $v=\pm 1/2$ or $\pm 2 \sin 2x$ Condone the omission of c
(d) $x^2+y^2=y$ or $x^2 = t = y - y^2$ $2x + 2y\, dy/dx = dy/dx$ $dt/dx = 2x$ $dt/dy = 1 - 2y$ $\frac{dy}{dx} = \frac{2x}{1-2y}$ Or M1 for an attempt to express x or y explicitly in terms of the other and to differentiate explicitly. A1 for the correct explicit equation and A1 for the correct differentiation. E1 for a correct and complete manipulation to obtain the result. Or Done by integration. M1 for separating the variables of the result . A1 for the correct integration condoning the omission of the arbitrary constant. E2 for including a constant and explaining that it can be given the value 0.	M1 A1 A1 E1 [4] Total 16	For an attempt at implicit differentiation wrt x or y or t Correct differentiation of x terms Ditto y terms Brackets must be seen where they are necessary Correct and complete manipulation and result

2 (i) $h = 2e^{-at}$ $\Rightarrow dh/dt = -2a\,e^{-at}$ $= -a\,h$ When $t = 0$, $h = 2\,e^0 = 2$ as required	M1 E1 E1 [3]	At least 2 e^{-at} correct plus something else Independent of M1.
Or $dh/dt = -ah \Rightarrow \int \frac{dh}{h} = \int -a\,dt$ $\Rightarrow \ln h = -at + c$ When $t = 0$, $h = 2 \Rightarrow \ln 2 = c$ $\Rightarrow \ln h = -at + \ln 2 \Rightarrow h = 2e^{-at}$	M1 E1 E1 [3]	Separating the variables and integrating. At least ln h and t. Condone the omission of c Finding c. Dependent on M1
(ii) $1 = 2\,e^{-10a}$ $\Rightarrow e^{-10a} = ½$ $\Rightarrow -10a = \ln(1/2)$ $\Rightarrow a = -\ln(1/2)/10 = 0.069(3\ldots)$	M1 DM1 A1 [3]	Substituting h=1 and t=10 taking lns or 0.07 or better.
(iii) $\frac{dH}{dt} = -b\sqrt{H}$ $\Rightarrow \int \frac{dH}{\sqrt{H}} = \int -b\,dt$ $\Rightarrow 2H^{1/2} = -bt + k$ When $t = 0$, $H = 1 \Rightarrow 2 = k$ $\Rightarrow 2\,H^{1/2} = -bt + 2$ $\Rightarrow \sqrt{H} = 1 - ½\,bt$ * A similar scheme can be applied to the integration of dt/dH	 M1 A1 B1 ft E1www. [4]	 Separating variables Condone the absence of k Evaluating their k. f.t. their solution; or substituting correct limits into their definite integrals.
(iv) When $t = 10$, $H = ½$ $\Rightarrow \sqrt{1/2} = 1 - 5b$ $\Rightarrow 5b = 1 - 1/\sqrt{2}$ $\Rightarrow b = (1 - 1/\sqrt{2})/5 = 0.05857\ldots$ $H = 0$ when $½\,bt = 1$ $\Rightarrow t = 2/b$ $= 34(.142\ldots)$ s	 M1 DM1 A1 M1 A1 [5] Total 15	 Substituting values in * Solving for b Either exact or 0.06 or better Solving 1−1/2 bt =0 Answers rounding to 34 secs.

3(a)(i) $\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}$ $=\frac{-2\sin 2\theta}{-\sin\theta}$ $=\frac{4\sin\theta\cos\theta}{\sin\theta}$ $=4\cos\theta$ * **or** Finding the Cartesian equation of the curve as in part (ii) (which may be given M1 A1 if it is referred to in part (ii)) and differentiating to give $dy/dx = 4x$ M1 A1 $= 4\cos\theta$ M1 E1	M1 A1 M1 E1www [4]	$\frac{their dy/d\theta}{their dx/d\theta}$ Allow $\frac{2\sin 2\theta}{\sin\theta}$ without other working but give final E0 $\sin 2\theta = 2\sin\theta\cos\theta$ used
(ii) $y = \cos 2\theta = 2\cos^2\theta - 1$ $= 2x^2 - 1$ **or** $dy/dx = 4\cos\theta = 4x \Rightarrow y = 2x^2 + c$ $x = 0$ $\theta = \pi/2$ $\cos 2\theta = -1 = y = c$	M1 A1 B1 [3] M1 A1	$\cos 2\theta = 2\cos^2\theta - 1$ used. Or any equivalent equation. Parabola with vertex at (0, -1) which must be indicated in some way c must be seen
(b) (i) $\cos 3\theta = 0$ $\Rightarrow 3\theta = \pi/2, 3\pi/2, 5\pi/2$ $\Rightarrow \theta = \pi/6, \pi/2, 5\pi/6$ $\Rightarrow x = \cos\theta = \sqrt{3}/2, 0, -\sqrt{3}/2$ So A is $(-\sqrt{3}/2, 0)$, B is $(\sqrt{3}/2, 0)$ For the correct coordinates as shown opposite given without any working, ie by the use of a graphical calculator, give B2, B2 for A is (-0.87, 0) and B is (0.87, 0)	M1 A1 A1www A1 www	s.o.i. θ = either $\pi/6$ or $5\pi/6$ accept 30° or 150° A is $(-\sqrt{3}/2, 0)$ Accept $x = -\sqrt{3}/2$ but not $(0, -\sqrt{3}/2)$ Accept -0.87. B is $(\sqrt{3}/2.\ 0)$ Accept as for A
(ii) $\cos(2\theta+\theta) = \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$ $= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$ $= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$ $= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$ $= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$ $= 4\cos^3\theta - 3\cos\theta$ So Cartesian equation is $y = 4x^3 - 3x$.	M1 M1 E1 B1 [4] Total 15	Compound angle formula correct Correct use of the double angle formulae using $\sin^2\theta + \cos^2\theta = 1$ and simplifying.

4 (i) $AC = \sqrt{\{2^2 + (-4)^2 + (-1)^2\}}$ $= \sqrt{21}$ or 4.58…	M1 A1 [2]	
(ii) $\vec{AB} = \begin{pmatrix}-4\\0\\-1\end{pmatrix}$ $\vec{AC} = \begin{pmatrix}-2\\4\\1\end{pmatrix}$ $\vec{AB}.\vec{AC} = (-4)\times(-2) + 0 \times 4 + (-1) \times 1$ ($= 7$	M1 A1	For vectors **AB** and **AC**, (accept **BA**, **CA**. Condone one slip in each vector), and for evaluating the scalar product.soi 7 must be seen.
$\cos BAC = \dfrac{7}{\sqrt{17} \times \sqrt{21}} = 0.3704..$	M1	Ft their vectors and their scalar product.
$\Rightarrow BAC = 68.25°$	A1 [4]	Or 68.3° or 1.19 radians
(iii) $\begin{pmatrix}2\\3\\-8\end{pmatrix}.\begin{pmatrix}-4\\0\\-1\end{pmatrix} = -8 + 0 + 8 = 0$	E1	-8+8 must be seen
$\begin{pmatrix}2\\3\\-8\end{pmatrix}.\begin{pmatrix}-2\\4\\1\end{pmatrix} = -4 + 12 - 8 = 0$	E1	-4+12-8 must be seen
so perpendicular to AB and AC Equation of plane: $2x + 3y - 8z = c$ At A, $2\times2 + 3\times0 - 8\times2 = c$ $\Rightarrow$ $c = -12$ $\Rightarrow$ $2x + 3y - 8z = -12$	M1 DM1 A1 [5]	substituting the coordinates of A, B or C or using $\underline{a}.\underline{n}$ If a vector equation is used give M1 for a correct form, DM1 for eliminating the two parameters and A1 for the result
(iv) H is $(1, 1, h)$ where $2\times1 + 3\times1 - 8\times h = -12$ $\Rightarrow 8h = 17$ $\Rightarrow h = 2.125$ (m)	M1 A1ft A1cao	Ft their equation from (iii)
<u>or</u> The equation of GH is $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}0\\0\\1\end{pmatrix}$	M1	
Meets AB where $2(1) + 3(1) - 8(\lambda) = -12$ $\lambda = 17/8$ and so h=2.125(m)	A1 ft A1 cao [3] Total 14	Ft their equation from (iii)

Examiner’s Report

2603 Pure Mathematics 3

General Comments

Candidates were given opportunities to perform well on this paper and many responded with high marks, a good number scoring in the range 70-75. A mark in this range was a notable achievement because there were some tricky points, which could catch out the strong as well as the weak. Some questions on the comprehension paper also gave able candidates as much trouble as the less able.

Generally there was a good spread of marks and only a small number of candidates scored fewer than 10 or 15.

The presentation of work was very varied; at the one extreme, immaculate scripts were a pleasure to mark, at the other, work was so muddled that it was difficult to decipher what the candidate was aiming to do. Poor notation was often a cause of this, brackets omitted, failure to be clear about which variable is being differentiated and with respect to which other variable, and attempts to integrate one variable with respect to a different variable. Perhaps the main cause of muddled work by many candidates, was the absence of any attempt to explain the logical progression of their work by the use of words or symbols; therefore, $\Rightarrow$, the correct use of the = sign, differentiating, etc. This was most evident in Section B, questions 3 and 4, and in Section A, question 1, part (d).

There was little evidence that candidates were short of time.

Comments on Individual Questions

Q.1

This question presented candidates with four standard procedures and generally they responded well. A very pleasing number scored full marks. Of course, there were errors from candidates who were clearly familiar with the methods involved.

(a) Most commonly, $-3x^2$ instead of $(-3x)^2$, or x instead of $3x$ etc., or perhaps just a careless sign error.

(b) Quite often the omission of brackets $Ax + B\ .\ (x - 1)$, although in many cases the subsequent working showed that the brackets had been implied. Errors sometimes occurred in the solution of correct equations in A, B and C.

(c) All too often, $\int \sin 2x dx = -1/2 \sin 2x$, or $2 \sin 2x$ or even $\frac{1}{2} \sin x$.

(d) Again, the omission of brackets, $1 - 2y.\frac{dy}{dx} = 2x$ was very common.

Of the four procedures in this question, implicit differentiation was the least familiar. This was a question where the notation of differentiation was crucial and where candidates who use the symbol $\frac{dy}{dx}$ as shorthand for 'differentiating' might not give convincing proofs of the stated result.

Q.2 Differential equations

(i) Many candidates attempted to solve the given differential equation rather than verify that $h = 2e^{-at}$ is a solution, as was suggested in the question. In the solution those candidates who omitted the arbitrary constant were restricted to one mark out of three because they were unable to use the initial condition. Those who included a constant often made the error $\ln h = -at + c \Rightarrow h = e^{-at} + e^c$. This was sometimes followed by a correct expression, $h = Ae^{-at}$, but without any justification. Those candidates who found the value of their constant as ln 2 were more likely to move correctly from $\ln h = -at + \ln 2$ to the required result.

Candidates who verified the result by differentiation were less likely to go wrong. Some poorer attempts at this question were a confused mixture of both methods. It is possible that some of these attempts failed because the candidates did not grasp that 'initially' implies $t = 0$.

(ii) This part was generally well done even by weaker candidates, most candidates making the correct substitutions and finding the value of a. The most likely error was $1 = 2\,e^{-10a} \Rightarrow \ln 1 = -10a \ln 2e$.

(iii) Only the most able candidates were able to solve this differential equation and they often produced immaculate solutions. For the rest there were many pitfalls. Some failed to separate the variables and attempted to integrate $\int -b\sqrt{H}\,dt$. Those who correctly obtained $\int \frac{dH}{\sqrt{H}} = \int -b\,dt$ often gave the LHS as $\ln\sqrt{H}$, or, if written as $\int H^{-\frac{1}{2}}dH$, integrated to $\frac{H^{\frac{1}{2}}}{2}$ or $\frac{H^{-\frac{3}{2}}}{-\frac{3}{2}}$. The RHS was sometimes given as $\frac{-b^2}{2}$ or $\frac{-b^2}{2}t$.

(iv) As in (ii) most candidates recovered to use the given result, substitute correctly and find the value of b, and very often go on to find the required time.

Q.3 Parametric equations

(a)(i) This part was generally very well done, candidates are familiar with the procedure and solutions were often correct.

(ii) Those candidates who thought to use the double angle formulae to express y in terms of cosθ or sinθ or both, usually managed to obtain the correct equation in some form, although just a few who chose the route $y = 1 - 2\sin^2\theta \Rightarrow \sin^2\theta = \frac{1-y}{2}$, sometimes then wrote $\left(\frac{1-y}{2}\right)^2 + x^2 = 1$. Candidates who obtained the equation usually made a correct sketch.

(b)(i) Most candidates started correctly with $\cos 3\theta = 0$. Those who solved this, most often gave only the solutions $3\theta = \pi/2$ and $3\pi/2$, thus omitting the solution $\theta = 5\pi/6$. This gave them the correct coordinates $(\sqrt{3}/2, 0)$ which they then identified as the point B. Many candidates then gave A correctly as the point $(-\sqrt{3}/2, 0)$ but failed in most cases to say why. Sometimes symmetry was mentioned but sometimes both $x = \pm\sqrt{3}/2$ were derived from $\theta = \pi/6$.

(ii) Again most candidates started correctly by expanding $\cos(2\theta + 0)$ as suggested in the question, but very many candidates were unable, successfully, to complete the question. Perhaps the most frequent breakdown came from dealing with $-\sin 2\theta \sin\theta = -2\sin\theta\cos\theta\sin\theta = -2\sin^2\theta\cos\theta$. A surprising number of candidates at this point used $2\sin^2\theta = 1 - \cos 2\theta$, thus returning to double angles, instead of using $\sin^2\theta + \cos^2\theta = 1$. If a Cartesian equation was given at the end of this question, it was usually the correct one.

Q.4 Vectors

This was a good question for very many candidates, they were familiar with the methods involved and carried them out accurately.

(i) Very well done by almost all candidates.

(ii) Most candidates obtained the correct angle but some did not state the scalar product clearly, as the question requested.

(iii) Almost all candidates knew that they had to show that $\begin{pmatrix}2\\3\\-8\end{pmatrix}.\begin{pmatrix}-4\\0\\-1\end{pmatrix}=0$ but a significant number did not state clearly that this scalar product was equal to $-8 + 0 + 8 = 0$, and similarly for $\begin{pmatrix}2\\3\\-8\end{pmatrix}.\mathbf{AC}=0$.

Those candidates approaching the Cartesian equation of the plane directly usually got the LHS correct but sometimes there was an error in sign on the RHS. Quite a large number of candidates still approach this question by starting with the vector equation of the plane and eliminating the parameters. Some did this successfully but others made errors with the algebra involved. In either event, of course, this procedure takes more time which could be better spent on another question.

(iv) The more able candidates did this question most successfully either by giving H the coordinates (1, 1, h) or writing down the equation of the line GH as $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}0\\0\\1\end{pmatrix}$ and substituting into the equation of the plane ABC, to find h or λ. The most common error was to use the equation of the perpendicular to the plane ABC through H instead of the vertical line through H. Some other candidates used the formula for the length of the perpendicular from a point to a plane, in effect, the same error.

Section B (Comprehension)

1&2 The first two questions presented candidates with no problem and were almost always done correctly.

3. Candidates who were confident with simple algebra were able to do this question successfully, but many got into a muddle with their attempt, and many solutions were difficult to follow through.

Candidates who used the substitution $\frac{1}{2}(a + b + c) = 50$, in dealing with the entries for A, were able to say 'and similarly for B and C'; but more candidates used the substitution $c = 100 - (a + b)$, given in the table, in which case a separate proof was needed for C where it was necessary to change the substitution to $a = 100 - (b + c)$.

Many candidates in this, and the remaining questions, quoted numerical examples from the text to 'justify' the general cases, not understanding that the questions they were answering were justifying steps in the process of moving from particular cases to general cases.

4. Many candidates quoted the sentence from the text on line 165, 'The figures in the last column in Table 12 are exactly 100 less than those in the last column of Table 11', the statement that they were being asked to justify in the general case. It was most difficult to follow many of the algebraic proofs given because of the lack of clarity about the starting point and the steps taken.

5. Although the first part of this question was generally answered correctly very few candidates were able to go any further. They failed to realise that rewriting the last column, as had been done with Table 5, would enable them to make the comments needed. Just a few made the necessary change deliberately and a few more came across the form needed accidentally in making the entries required by part (i). Most other candidates simply repeated comments from the text.